Mat 120 Exam 3
March 9, 2011

Name: KEYS

Show all your work.

Remarks:
- While solving the problems try to apply standard tools introduced in class. You are more than welcome to use your own creative ideas during the exam, but make sure that you justify properly your arguments.
- You can use back of every page as a scratch paper (nothing on the back of a page will be graded).
- Please try to illustrate how you obtained solution, instead of simply providing an answer. You will not get any credit for simply stating the answer, even if your answer is the correct one.
- You are allowed to use calculators, but make sure that you indicated clearly what equation you solve with help of your calculator.

Start time 9.20 a.m. End time 10.30 a.m.

MAX 75

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1) (5 points) If you paid $30 to a loan company for the use of $1000 for 60 days, what annual rate of interest did they charge? Assume simple interest.

\[ A = P(1 + rt) \]
\[ \frac{A}{P} = 1 + rt \]
\[ \frac{A}{P} - 1 = rt \]
\[ r = \frac{A - P}{Pt} \]
\[ r = \frac{1030 - 1000}{1000 \cdot \frac{2}{12}} \]
\[ = 0.18 \]
\[ \text{Answer} \ 18\% \]

2) (5 points) What is the purchase price of 50-day T-bill with a maturity value of $1000 that earns an interest rate of 5.53%? Assume simple interest.

\[ A = P(1 + rt) \]
\[ P = \frac{A}{1 + rt} \]
\[ P = \frac{1000}{1 + 0.0553 \cdot \frac{50}{360}} = 992.38 \]
3) (10 points)

<table>
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<th>Principal</th>
<th>Commission</th>
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<tr>
<td>Under $3000</td>
<td>$32 + 1.8% of principal</td>
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<tr>
<td>$3000 – $10000</td>
<td>$56 + 1% of principal</td>
</tr>
<tr>
<td>Over $10000</td>
<td>$106 + 0.5% of principal</td>
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An investor purchased 215 shares at $45.75 a share, holds the stock for 300 days, and then sells the stock for $51.90 a share. Use the above commission schedule to find the annual rate of interest earned.

Total initial investments:

\[
\frac{215 \cdot 45.75 + 56 + 0.01 \cdot 9836.25}{9836.25} = 9990.61
\]

Net proceeds (after sale)

\[
\frac{215 \cdot 51.90 - 106 - 0.005 \cdot 11158.50}{11158.50} = 10996.7075
\]

\[
r = \frac{A - P}{P \cdot t} = \frac{10996.7075 - 9990.61}{9990.61 \cdot \frac{360}{360}} = 0.12085
\]

Answer 12.085%

4) (5 points) How long will it take $6000 to grow to $8600 if it is invested at 9.6% compounded continuously.

\[
A = Pe^{rt}
\]

\[
\frac{A}{P} = e^{rt}
\]

\[
\ln \left( \frac{A}{P} \right) = rt
\]

\[
\ln \frac{A}{P} = rt
\]

\[
t = \frac{1}{r} \ln \frac{A}{P}
\]

\[
t = \frac{1}{0.096} \ln \frac{8600}{6000} = 3.75
\]

Answer 3.75 years
5) (5 points) If the population in a particular country is growing at 1.7% compounded continuously, how long will it take the population to double?

Let \( P_0 \) be initial population

Then growth model \( P = P_0 e^{rt} \)

Therefore

\[
2 P_0 = P_0 e^{rt} \quad t = \frac{\ln 2}{0.017} = 40.77
\]

\[
2 = e^{rt} \quad \ln 2 = rt \quad \text{Answer} \ 40.77 \text{ years}
\]

\[
t = \frac{\ln 2}{r}
\]

6) (5 points) What annual nominal rate compound monthly has the same annual percentage yield as 7% compounded continuously?

\[
\text{APY} = e^r - 1
\]

\[
\text{APY} = e^{0.07} - 1 = 0.07251
\]

\[
\text{APY} = (1 + \frac{r}{m})^m - 1 \quad \text{solve for} \ r
\]

\[
\text{APY} + 1 = (1 + \frac{r}{m})^m \quad r = \left(0.07251 + 1\right)^{\frac{12}{12}} - 1
\]

\[
(\text{APY} + 1)^{\frac{12}{12}} = 1 + \frac{r}{m} \quad r = 0.0702
\]

\[
r = (\text{APY} + 1)^{\frac{12}{12}} - 1\]

\[
\text{Answer:} \ 7.02\% \]

7) (5 points) One investment pays 10% compounded quarterly and another pays 9.5% compounded monthly. Which investment is more profitable? You have to base your answer on annual percentage yield.

\[ \text{APY} = \left(1 + \frac{r}{m}\right)^m - 1 \]

\[ \text{APY}_1 = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.10381 \]

\[ \text{APY}_2 = \left(1 + \frac{0.095}{12}\right)^{12} - 1 = 0.09925 \]

Answer: 10% quarterly is better.

8) (5 points) A zero coupon bond with a face value of $20000 matures in 10 years. What should the bond be sold for now, if its rate of return is to be 4.194% compounded annually?

\[ A = P \left(1 + \frac{r}{m}\right)^{mt} \]

\[ P = \frac{A}{\left(1 + \frac{r}{m}\right)^{mt}} = \frac{20000}{\left(1 + 0.04194\right)^{10}} = 13261.81260 \]
9) (5 points) In order to accumulate enough money for a down payment on a house, a couple deposits $300 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much money will be in the account in 5 years?

\[
FV = \frac{PMT \left( (1+i)^n - 1 \right)}{i}
\]

\[
FV = 300 \frac{(1 + 0.06/12)^{60} - 1}{0.06/12} = 20931.00915
\]

10) (10 points) Beginning in January, a person plans to deposit $100 at the end of each month into an account earning 6% compounded monthly. Each year taxes must be paid on the interest earned during that year. Find the interest earned during each year for the first 3 years.

Let's compute future value in 1 year.

\[
FV = 100 \cdot \frac{(1 + 0.06/12)^{12} - 1}{0.06/12} = 1233.55624 \approx 1233.56
\]

Since he deposited only 12 $100, the interest is $1233.55624 - 1200 = 33.55624$ (1st year)

Next year his 12 deposits also earned $33.55624$ interest, plus interest on $1233.55624$, which is $I = P \left( (1+i)^n - 1 \right) - P$

\[
I = 1233.55624 \left( (1 + 0.06/12)^{12} - 1 \right) = 1233.55624 = 76.08305
\]

Therefore, total interest for year 2 is $33.55624 + 76.08305 = 109.64$

At the beginning of the 3rd year he had in the account

\[
FV = 100 \cdot \frac{(1 + 0.06/12)^{24} - 1}{0.06/12} = 2543.19552
\]

His 12 deposits in year 3 earned him $33.56$, plus interest on $2543.19552$

\[
I = 2543.19552 \left( (1 + 0.06/12)^{12} - 2543.19552 \right) = 156.85874
\]

So total $190.41$ (year 3)
11) (5 points) American General offers a 10-year ordinary annuity with a guaranteed rate of 6.65% compounded annually. How much should you pay for one of these annuities, if you want to receive payments of $5000 annually over the 10-years period?

\[
P_{\text{V}} = \frac{PMT}{i} \left(1 - \left(1 + \frac{i}{n}\right)^{-n}\right)
\]

\[
P_{\text{V}} = 5000 \left(1 - \left(1 + \frac{0.0665}{10}\right)^{-10}\right) = 35695.18
\]

12) (10 points) A person purchased a $145000 home 10 years ago by paying 20% down and signing a 30-year mortgage at 7.9% compounded monthly. Interest rates have dropped and the owner wants to refinance the unpaid balance by signing a new 20-year mortgage at 5.5% compounded monthly. How much interest will refinancing save?

Let's compute unpaid balance (payment first)

\[
PMT = \frac{PV \cdot i}{n} = 145000 \cdot 0.8 \left(1 + \frac{0.079}{12}\right) = 843.09
\]

Remaining balance

\[
= 843.09 \cdot \left(1 + \frac{0.079}{12}\right)^{-240} = 101549.82
\]

Compute interest under 7.9% (20 years) 843.09 \cdot 240 - 101549.82 = 100872.31

Compute payment under 5.5% (20 years)

\[
PMT = \frac{101549.82 \cdot 0.055/12}{1 - \left(1 + \frac{0.055}{12}\right)^{-240}} = 576.59
\]

13) (3 points) BONUS question. Why are you taking Analytical Methods? (Answers like "because I have to" will earn you zero points)

Compute interest under 5.5% (20 years) 576.59 \cdot 240 - 101549.82 = 36831.47

Total savings = 100872.31 - 36831.47 = 64040.84