Mat 120 Exam 4
April 6, 2011

Name: KEYS

Show all your work. No calculators. If you need to compute something you can ask me.

Remarks:
- While solving the problems try to apply standard tools introduced in class. You are more than welcome to use your own creative ideas during the exam, but make sure that you justify properly your arguments.
- You can use back of every page as a scratch paper (nothing on the back of a page will be graded).
- Please try to illustrate how you obtained solution, instead of simply providing an answer. You will not get any credit for simply stating the answer, even if your answer is the correct one.
- You are allowed to use calculators, but make sure that you indicated clearly what equation you solve with help of your calculator.

Start time 9.20 a.m. End time 10.30 a.m.

MAX 55

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1) (10 points) A ship using sound-sensing devices above and below water recorded a surface explosion 6 seconds sooner by its underwater device that its above-water device. Sound travels in air at 1100 feet per second and in seawater at 5000 feet per second. How long did it take each sound wave to reach the ship? How far was the explosion from the ship? (You have to setup a system of linear equations - You will not get a credit for any other solutions)

Let \( x \) - time it took the explosion (sound wave) to reach the underwater device.

\( y \) - time it took the explosion (sound wave) to reach the above-water device.

Then \( y - 6 = x \)

\[ y = \frac{1100}{5000} \text{ distance from explosion to the ship} \]

\[ x = \frac{5000}{5000} \text{ distance from explosion to the ship} \]

\[ y - 6 = x \]

\[ y = \frac{30000}{5900} = \frac{100}{13} \Rightarrow x = \frac{22}{13} \]

Now for \( \frac{100}{13} \) units.

2) (5 points) Solve using augmented matrix method.

\[
\begin{bmatrix}
1 & -2 & 5 \\
-2 & 4 & -10 \\
1 & -2 & 5 \\
0 & 0 & 0
\end{bmatrix}
\]

\[ 2R_1 + R_2 \Rightarrow R_2 \]

\[
\begin{bmatrix}
1 & -2 & 5 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\( x_1 - 2x_2 = 5 \)

\( x_2 = t \), \( x_1 = 5 + 2t \), \( \forall t \in \mathbb{R} \)
3) (5 points) Solve using Gauss-Jordan elimination. Write your answer in the form \( x_1 = f(t), \ x_2 = g(t), \ x_3 = t \) for any real number.

\[
2x_1 + 4x_2 - 6x_3 = 10 \\
3x_1 + 3x_2 - 3x_3 = 6
\]

\[
\begin{bmatrix}
2 & 4 & -6 \\
3 & 3 & -3
\end{bmatrix}
\begin{bmatrix}
1 & 2 & -3 \\
0 & -1 & 2
\end{bmatrix}
\]

\[
R_1 - \frac{1}{2} \rightarrow R_1
\]

\[
\begin{bmatrix}
1 & 2 & -3 \\
3 & 3 & -3
\end{bmatrix}
\begin{bmatrix}
1 & 2 & -3 \\
0 & 1 & -2
\end{bmatrix}
\]

\[
R_1 - 2R_2 \rightarrow R_1
\]

\[
\begin{bmatrix}
1 & 2 & -3 \\
3 & 3 & -3
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -2
\end{bmatrix}
\]

\[
R_2 - R_1 \rightarrow R_2
\]

\[
x_1 + x_3 = -1 \\
x_2 - 2x_3 = 3
\]

\[
x_1 = -1 - t, \ x_2 = 3 + 2t, \ x_3 = t \\
\forall t \in \mathbb{R}.
\]

4) (10 points) A chemical manufacturer wants to lease a fleet of 24 railroad tank cars with a combined carrying capacity of 520 thousand gallons. Tank cars with three different carrying capacities are available: 8 thousand gallons, 16 thousand gallons and 24 thousand gallons. How many of each type of tank car should be leased? What is the minimum number of 24 thousand gallons tanks could be leased?

Let \( x \) - \# of cars of 8000 gallon capacity to lease.

\[
y = \# \quad 16000
\]

\[
z = \# \quad 24000
\]

\[
x + y + z = 24
\]

\[
8x + 16y + 24z = 520 \leq \text{capacity equation.}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 24 \\
8 & 16 & 24 & 520
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 24 \\
0 & 1 & 41
\end{bmatrix}
\]

\[
R_1 - R_2 \rightarrow R_1
\]

\[
\begin{bmatrix}
1 & 0 & -1 & -17 \\
0 & 1 & 41
\end{bmatrix}
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & 41
\end{bmatrix}
\]

\[
x - z = -17 \\
y + 2z = 41
\]

\[
x = -17 + z \geq 0 \Rightarrow z \geq 17 \\
y = 41 - 2z \geq 0 \Rightarrow z \leq \frac{41}{2} \leq 20.
\]
Therefore \( 17 \leq z \leq 20 \)

\[
\begin{align*}
\text{let } & \quad z = t \\
\text{then } & \quad x = -17 + t \\
\text{so } & \quad y = 41 - 2t
\end{align*}
\]

The minimum number of 24,000 gallons cars to lease is 17.
5) (5 points) Examine the product of the two matrices to determine if each is the inverse of the other.

\[
A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[AB = \begin{bmatrix} 1 & 0 & -2 \end{bmatrix} \leq \text{already enough to conclude that } A \text{ is not an inverse of } B\]

6) (5 points) Find the inverse of the following matrix, if it exists.

\[
A = \begin{bmatrix} -4 & 3 \\ -5 & 4 \end{bmatrix}
\]

\[
\begin{bmatrix} -4 & 3 \\ -5 & 4 \end{bmatrix} \rightarrow
\begin{bmatrix} 1 & -3/4 \quad -1/4 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[R_1 + \frac{3}{4} R_2 \rightarrow R_2
\]

\[R_2 + 5 R_1 \rightarrow R_2
\]

\[\begin{bmatrix} 1 & -3/4 \quad -1/4 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[4R_2 \rightarrow R_2
\]

\[\begin{bmatrix} 1 & 0 & -4/3 \\ 0 & 1 & -5/4 \end{bmatrix} \text{ Inverse.}
\]
7) (5 points) Write each system as a matrix equation and solve using inverse

\[
\begin{align*}
2x_1 + x_2 &= -1 \\
x_1 + x_2 &= -2
\end{align*}
\]

\[
\begin{bmatrix}
2 & 1 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
-1 \\
-2
\end{bmatrix}
\]

\[
R_2 - R_1 \rightarrow R_2
\]

\[
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & \frac{1}{2} & -\frac{1}{2} & 1
\end{bmatrix}
\begin{bmatrix}
1 & -1 \\
-1 & 2
\end{bmatrix}
= \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\]

\[
\frac{1}{2}R_1 \rightarrow R_1
\]

\[
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 1 & -1 & 2
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 1 & -1 \\
0 & 1 & -1 & 2
\end{bmatrix}
\]

\[
x_1 = 1
\]

\[
x_2 = -3.
\]

8) (5 points) Solve for \( x_1 \) and \( x_2 \)

\[
\begin{bmatrix}
3 & 1 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
\begin{align*}
3 & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =
\begin{bmatrix}
7 \\
8
\end{bmatrix}
\end{align*}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 1 & -1 \\
0 & 1 & -2 & 3
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
2 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\begin{align*}
x_1 =
\begin{bmatrix} 1 -1 \\ -2 3 \end{bmatrix} =
\begin{bmatrix} -4 \\ -15 \end{bmatrix}
\end{align*}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & \frac{1}{2} & -\frac{1}{2} & 1
\end{bmatrix}
\begin{bmatrix}
\begin{align*}
x_2 =
\begin{bmatrix} -2 3 \\ 7 \end{bmatrix} =
\begin{bmatrix} -4 \\ 15 \end{bmatrix}
\end{align*}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 1 & -2 & 3
\end{bmatrix}
\]

\[
x_1 = -4
\]

\[
x_2 = 15.
\]
9) (5 points) Find \(AB\) and \(BA\), if they exist

\[A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}\]

\[AB = (2 \times 3) (2 \times 2) \text{ does not exist}\]

\[BA = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -6 & 7 & -11 \\ 4 & 18 & -4 \end{bmatrix}\]

10) BONUS Problem (5 points) Two airplanes travel toward each other from cities that are 780 km apart at speeds of 190 km/h and 200 km/h. They left at the same time. In how many hours will they meet? (You have to set up a system of equations and solve it. You will not get any points for any other solutions)

\[x - \text{distance traveled by plane with speed 200 km/h}
\]

\[y - \text{distance traveled by the other plane.}\]

\[\text{City A} \quad \text{City B} \quad 780\]

\[x + y = 780 \quad \text{1}\]

They traveled for the same time

\[\frac{x}{200} = \frac{y}{190} \Rightarrow x = \frac{200}{190} y \Rightarrow \frac{200y}{190} + y = 780\]

\[390y = 780 \cdot 190 \]

\[y = \frac{780 \cdot 190}{390} = \frac{380}{190}\]

They meet in \(\frac{380}{190} = 2\) hours.